Contextual Budget Allocation for Food Rescue Volunteer Engagement

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Abstract

Volunteer-based food rescue platforms tackle food waste by matching surplus food to communities in need. These platforms face the dual problem of maintaining volunteer engagement and maximizing the food rescued. Existing algorithms to improve volunteer engagement exacerbate geographical disparities, leaving some communities systematically disadvantaged. We address this issue by extending restless multi-armed bandits, a model of decision-making which allows for stateful arms, to incorporate context-dependent budget allocation. By doing so, we can allocate higher budgets to communities with lower match rates, thereby alleviating geographical disparities. To tackle this problem, we develop an empirically fast heuristic algorithm. Because such an algorithm can achieve a poor approximation when active volunteers are scarce, we design the Mitosis algorithm, which is guaranteed to compute the optimal budget allocation. Empirically, we demonstrate that our algorithms outperform baselines on both synthetic and real-world food rescue datasets, and show how our algorithm achieves geographical fairness in food rescue.

1 Introduction

The world wastes up to 40% of our food globally, translating to over 1.3 billion tons annually, while 1 in 7 people struggle to secure enough food every day [Coleman-Jensen *et al.*, 2018; Conrad *et al.*, 2018]. With their appearance in over 100 cities worldwide, food rescue platforms (FRP) receive safe, edible food donations from businesses like restaurants ("donors") and distribute them to organizations serving low-resource communities ("recipients"). Our partner organization, Food Rescue X[1], is a large FRP with operations in over 25 different cities across the US. FRPs are able to scale due to volunteers, who transport food from donors to recipients. Essentially, volunteers claim "rescues" from an FRP's mobile app. After claiming the rescue, the app instructs them where to pick up and drop off the donation.

[1]Real name blinded.

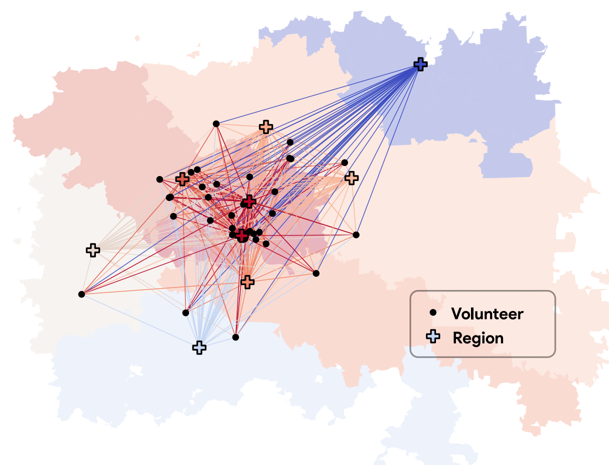

Figure 1: The picture shows volunteers and donation regions in real food rescue database. Region color indicates the richness of volunteer resource. Connected lines indicates how volunteers and real-time donation tasks are matched by food-rescue platforms.

The inclusion of volunteers in FRPs brings about inherent uncertainty due to changing volunteer behavior. Volunteer engagement is critical to FRP success, so FRPs have an urgent need to engage their volunteers while maximizing the amount of food rescued. A few studies have developed algorithms to improve volunteer engagement on FRPs by dynamically notifying volunteers about rescue trips [Shi *et al.*, 2021, 2024; Raman *et al.*, 2024]. However Shi *et al.* [2021] showed that such algorithms can backfire because they result in severe geographical disparity in food rescue outcomes. In some regions such as downtown, the algorithm enjoyed almost 90% completion rate, while in some outer suburbs, the completion rate dropped to 40%.

In our work, we study how to maintain volunteer engagement while combatting geographical disparities. The challenge is that volunteer behaviors evolve over time in response to notification patterns. To tackle this issue, we model food rescue volunteer engagement as a restless multi-armed bandit (RMAB) problem, a common model for online resource allocation [Mate *et al.*, 2020; Raman *et al.*, 2024]. We extend this model to incorporate geographical disparities with a *context*, which corresponds to geographic information for each rescue trip. We then set notification quotas for different regions so certain regions with scarce volunteers have higher budgets. Such an approach allows for flexibility in notifications without sacrificing overall performance.

We make the following contributions: (1) We propose the *Contextual Budget Bandit* problem, which extends RMABs to situations with context-dependent budget allocations. Such a problem is motivated by applications in food rescue, but can also model problems in domains such as digital farming and peer review (see Appendix E). (2) We develop the COcc, a fast, empirically approximation algorithm which provides an upper bound to Contextual Budget Bandit. We characterize cases where it fails with a constant factor; (3) We design the Mitosis algorithm which is guaranteed to compute the optimal budget allocation; and (4) We empirically demonstrate that our algorithms improve upon baselines with synthetic and real-world food rescue datasets.

2 Preliminary Background

A Restless Multi-Armed Bandit (RMAB) is defined by:

$$\langle N, \mathcal{S}, \mathcal{A}, \{r_i\}_{i\in[N]}, \{P_i\}_{i\in[N]}\rangle.$$

Each arm $i \in [N] := \{1, 2, \ldots, N\}$ is an independent Markov Decision Process, with state space $\mathcal{S}_i = \{0, 1\}$ and binary action space $\mathcal{A}_i = \{0, 1\}$. Action 0 corresponds to idling the arm while action 1 corresponds to pulling the arm. The reward function for each arm $r_i : \mathcal{S}_i \times \mathcal{A}_i \to \mathbb{R}$ maps state-action pairs to a reward. P_i is the transition kernel for each arm i. The overall system state at time t is $\mathbf{s}^t = (s_1^t, s_2^t, \ldots, s_N^t)$, and the decision maker selects action $\mathbf{a}^t = (a_1^t, a_2^t, \ldots, a_N^t)$ subject to a budget constraint:

$$\sum_{i\in[N]} a_i^t \leq B, \quad \forall\, t = 1, 2, \ldots,$$

which limits the number of arms that can be pulled in every time step. The objective is to design a policy that maps the current state $\mathbf{s}^t$ to an action vector $\mathbf{a}^t$ that maximizes the average reward over all arms and over an infinite time horizon.

A widely-adopted approach for tackling the computational complexity inherent in RMABs is the Whittle Index Policy. The *Whittle Index* for each arm i's state $s_i \in \mathcal{S}_i$ is:

$$w_i(s_i) = \min_w \{w | Q_{i,w}(s_i, 0) = Q_{i,w}(s_i, 1)\},$$

$$Q_{i,w}(s_i, a_i) = -wa_i + r_i(s_i, a_i) + \gamma \sum_{s'} P_i[s_i, a_i, s'] V_{i,w}(s')$$

$$V_{i,w}(s') = \max_a Q_{i,w}(s', a)$$

$Q_{i,w}(s_i, a_i)$ represents the expected future reward for playing action a_i, given a penalty w for pulling an arm. Under the crucial condition of **indexability**—which requires that the set of states where it is optimal to activate an arm decreases monotonically as the subsidy w increases—the Whittle index is well-defined and interpretable as the marginal value of activating an arm. At each time step, the *Whittle Index Policy* pulls the B arms with the highest Whittle Indices. In this way, it decouples the multi-armed problem into a collection of single-arm problems. The Whittle index policy is asymptotically optimal under regularity conditions as the number of arms goes to infinity [Gittins *et al.*, 2011].

3 Contextual Budget Bandit

Because traditional methods to maintain volunteer engagement can lead to geographical disparity [Shi *et al.*, 2021], we pursue an intuitive solution where we allocate different notification budget to different regions. To do this, we need to augment the standard RMAB model with variability in transition and reward across time, and the flexibility to adjust budget accordingly. In this section, we will introduce the Contextual Budget Bandit model and multiple algorithms for it.

3.1 The Contextual Budget Bandit Model

A Contextual Budget Bandit (CBB) is defined by the tuple

$$\langle N, \mathcal{S}, \mathcal{A}, K, \{r_i^k\}_{i\in[N],k\in[K]}, \{P_i^k\}_{i\in[N],k\in[K]}, \mathcal{F}\rangle.$$

Departing from the standard RMAB model, we introduce the $[K] = \{1, 2, \ldots, K\}$ (finite) **contexts**. A Borel measure $\mathcal{F}$ on $[K]$ specifies the distribution over these contexts, which is known by the decision maker. At each time step, a new context is sampled with respect to $\mathcal{F}$ and globally applies to all arms. r_i^k, $P_i^k, \forall k \in [K]$ are the reward function and the transition probability kernels *specific to context* k. $\mathcal{F}$ and $\{P_i^k\}_{i\in[N],k\in[K]}$ are independent.

Definition 3.1 (Context Specific Budget Constraint)**.** A policy is said to satisfy *Context Specific Budget Constraint* if the number of arms pulled at each time step is constrained by a budget B_k contingent on context, while the expected budget usage is still bounded by B:

$$\sum_{i\in[N]} a_i^t \mathbb{I}(k^t = k) \leq B_k, \quad \forall t, k, \quad \text{and} \quad \mathbb{E}_{k\sim\mathcal{F}} B_k \leq B.$$

A policy π for CBB (i) pre-specifies budget allocation $\vec{B}$ and (ii) maps the current states of all arms and the context k to an action vector: $\pi : \{\mathcal{S}, [K]\} \mapsto \mathcal{A}$. The objective is then to maximize the expected average reward across timesteps, where the expectation is taken over contexts:

$$\lim_{T\to\infty} \mathbb{E}_{\mathbf{a}\sim\pi,k\sim\mathcal{F}} \Big[\frac{1}{T}\sum_{t=1}^{T}\sum_{i\in[N]} r_i^k(s_i^t, a_i^t)\Big].$$

The Whittle Index Policy for standard RMAB satisfies the above Context Specific Budget Constraint (Definition 3.1), because it uses a uniform budget for each context $\vec{B} = (B, \ldots, B)$. However, its performance can be arbitrarily bad:

Theorem 1. For a CBB, denote the Whittle Index Policy's reward as $\mathcal{R}^{\text{VanillaWhittle}}$, and the optimal policy that satisfies context-specific budget constraint as $\mathcal{R}^{\text{ContextOpt}}$. There exists an instance where,

$$\frac{\mathcal{R}^{\text{ContextOpt}}}{\mathcal{R}^{\text{VanillaWhittle}}} \to \infty, \qquad \text{as } N \to \infty.$$

3.2 Contextual Occupancy Index (COcc) Policy

Having established that the vanilla Whittle Index policy could perform arbitrarily bad in CBB, we now develop an efficient heuristic algorithm, the COcc. First, we introduce the *occupancy measure*.

Definition 3.2. The occupancy measure μ of a (possibly randomized) policy π in CBB is the average visitation probability to a state-action-context tuple $(s, a; k)$:

$$\mu_i(s, a; k) := \mathbf{Pr}\left[s_i = s, a_i = a; k\right], \forall i \in [N].$$

We next show how we can formulate the problem of maximizing the stationary reward as a linear program (LP) over occupancy measures.

Definition 3.3. For a given contextual-RMAB instance, its *occupancy-measure LP* is

$$\max_{\mu} \sum_{i\in[N]} \sum_{k\in[K]} \sum_{s_i,a_i} \mu_i(s_i, a_i; k) r_i(s_i, a_i; k) \tag{1}$$

$$s.t.\ f'_k \left[\sum_{k\in[K]} \sum_{s_i,a_i} P[s_i \to s'_i | a_i, k] \mu_i(s_i, a_i; k) \right] \tag{2}$$

$$= \sum_{a_i} \mu_i(s'_i, a_i; k'), \forall k', s'_i, i \tag{3}$$

$$\sum_{k\in[K]} \sum_{a_i,s_i} \mu_i(s_i, a_i; k) = 1, \forall i \in [N] \tag{4}$$

$$\sum_{k\in[K]} \sum_{i} \sum_{s_i} \mu_i(s_i, 1; k) \le B \tag{5}$$

$$\mu_i(s_i, a_i, k) \ge 0, \forall i, s_i, a_i, k. \tag{6}$$

Definition 3.4 (Adapted from [Xiong *et al.*, 2022])**.** Given the optimal solution $\mu^\star(\cdot, \cdot; k)$ to the occupancy-measure LP, the Contextual Occupancy **Soft Budget** Policy π^{soft} pulls an arm i in state s_i and context with probability $\chi_i^\star(s_i, k)$, where

$$\chi_i^\star(s_i, k) = \frac{\mu_i^\star(s_i, 1; k)}{\mu_i^\star(s_i, 0; k) + \mu_i^\star(s_i, 1; k)}.$$

The Contextual Occupancy **Soft Budget** Policy is not immediately applicable because the budget constraint in Equation 5 is only a relaxed version. Thus, denoting the occupancy-measure LP's objective value as $\overline{\mathcal{R}}$, we have

$$\mathcal{R}^{\text{VanillaWhittle}} \le \mathcal{R}^{\text{ContextOpt}} \le \overline{\mathcal{R}}. \tag{7}$$

The Contextual Occupancy **Soft Budget** Policy can achieve a high reward because it can shift budget across time — by saving up budget at bad context and using them when context is good. Guided by this insight, we introduce the Contextual Occupancy Index Policy (COcc) that mimics the Contextual Occupancy **Soft Budget** Policy, but further satisfies a context-dependent budget constraint for a set of budgets $\vec{B}$:

$$B_k = \frac{1}{f_k} \sum_{i,s_i} \mu_i^\star(s_i, 1; k), \qquad \forall k$$

where $\mu^\star$ are optimal solutions from occupancy-measure LP.

Given a context k, to determine which arms to pull, we tend to the dual of the occupancy-measure LP[2]. Let $V_i(s_i, k), \forall i, s_i, k$ be the Lagrangian for constraint (2, 3). Let $\nu_i, \forall i$ be the Lagrangian multiplier for (4), and ρ for (5). The dual of the occupancy-measure LP is

$$\min_{V,\rho,\nu} \sum_{i\in[N]} \nu_i + \rho B \tag{8}$$

$$s.t.\ V_i(s_i, k) + \nu_i \ge r_i(s_i, a_i; k) - \rho \mathbb{I}\{a_i = 1\} \tag{9}$$

$$+ \sum_{s'_i,k'} V_i(s'_i, k') P[s_i \to s'_i | a_i, k], \forall s_i, a_i, k \tag{10}$$

$$\rho \ge 0 \tag{11}$$

The solution of the dual of the occupancy-measure LP partitions every arm's state-context pair (s_i, k) into three sets, E_0, E_1 and E_{01} where respectively, the optimal action is positive, negative or some randomization.[3]

The optimal Lagrangian multiplier $\rho^\star$ in the dual problem can be interpreted as an extra cost for taking the positive action ($a_i = 1$). Under indexability, as the cost $\rho^\star$ increases, the set of state-context (s_i, k) in which negative action is optimal (E_0) increases monotonically. Thus, we define the *Contextual Whittle Index* for each state-context pair (s_i, k) of each arm, denoted as $\rho_i^\star(s_i, k)$, as the least value of the cost ρ such that negative action is optimal:

$$\rho_i^\star(s_i, k) := \sup\{\rho : (s_i, k) \in E_0\}.$$

With this, we are ready to formally define our COcc.

Definition 3.5 (The <u>C</u>ontextual <u>**Occ**</u>upancy Index (COcc) Policy)**.** At each time step, given context k, the COcc pulls top-B_k arms that have the highest positive Contextual Whittle Index, where

$$B_k = \frac{1}{f_k} \sum_{i,s_i} \mu_i^\star(s_i, 1; k),$$

with $\mu^\star$ being optimal solutions from occupancy-measure LP.

Standard RMAB problems correspond to B_k being the same across all k, and in such a situation,the COcc is equivalent to the Whittle Index Policy, and is asymptotically optimal.[4] However, the COcc is not optimal for CBB:

Theorem 2. The COcc's asymptotic approximation ratio compared to $\mathcal{R}^{\text{ContextOpt}}$ is bounded above by $\frac{5}{6}$.

The source of the suboptimality comes from when there are more than one contexts. The proof in Appendix C presents an original mathematical framework for asymptotic analysis.

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[2] detail of obtaining the dual is in Appendix B

[3] To avoid uninteresting pathologies, assume every pure policy gives rise to a Markov chain with one recurrent class, then the randomization set E_{01} need not contain more than one object [Gittins *et al.*, 2011].

[4] The asymptotic notion is usually to repeat all arms of an RMAB instance infinitely, along with the budget for the same repeats.

3.3 Solving Optimal Budget Allocation Using Multi-Armed Bandit Algorithm

The COcc is suboptimal because it fails to determine the optimal budget allocation $\vec{B}$. Yet Contextual Whittle Index can still be used given a budget allocation.

Definition 3.6. A *Flexible Budget Allocation COcc* determines a budget allocation $\vec{B} \in \mathcal{B}_0 := \{\vec{B} \in \mathbb{N}^K : \sum_{k=1}^K B_k \leq B\}$ given a total budget B and context probabilities $\vec{f}$, and pulls the B_k arms with the highest Contextual Whittle Index, $\rho_i^*(s_i, k)$.

Our goal is thus to find the optimal budget combination $\vec{B}^*$ from the set of feasible budget allocations $\mathcal{B}_0$. Within the class of Flexible COcc(s), each budget allocation $\vec{B}$'s reward can be evaluated according to an Oracle function:

Definition 3.7 (Oracle)**.** The Oracle is a randomized simulation procedure that, given a budget allocation $\vec{B} \in \mathbb{N}^K$, runs the Flexible COcc using $\vec{B}$ and returns the resulting reward. It has two parameters:

- `Epochs`: number of times simulation is repeated.
- `T`: the length of each simulation run.

Oracle gives us an estimate of policy performance, but it is slow. Meanwhile, we can obtain upperbounds for each budget allocation by inserting the following constraint into the occupancy-measure LP in Definition 3.3:

$$\frac{1}{f_k} \sum_{i,s_i} \mu_i(s_i, 1; k) = B_k \quad \forall B_k \in \vec{B}. \tag{12}$$

Given budget allocation $\vec{B}$, let $\texttt{LP}(\vec{B})$ be the optimal value of occupancy-measure LP with constraint (12) inserted.

We next design two algorithms to find the best budget allocation by querying the Oracle and LP for a small subset of budget combinations, avoiding an exponential enumeration.

The Branch And Bound Algorithm Since $\texttt{LP}(\vec{B}) \geq \text{Oracle}(\vec{B}), \forall \vec{B}$, we design a Branch And Bound approach to efficiently search over the feasible solution space: it recursively splits the search region into smaller subregions and prunes subregions if its LP-based upperbound is lower than another's actual reward. Although Branch And Bound is still NP-hard in the worst case, it provides a systematic way to efficiently search. We provide the pseudocode (Algorithm 2) in appendix F.

The Mitosis Algorithm While Branch And Bound already cuts down the search tree dramatically compared to brute force search, it still calls too many costly Oracle evaluations on large-scale problems. To address this issue, we develop the following multi-armed bandit (MAB) framework which allows for more nuanced speed-accuracy trade-off for the evaluation of budget allocations:

Definition 3.8 (MAB on top of Contextual-RMAB)**.** We define an associated *Multi-Armed Bandit (MAB) problem* by identifying each arm with a vector $\vec{B} \in \mathcal{B}_0$. Pulling arm $\vec{B}$ invokes the fast oracle $\text{Oracle}_{\text{small}}(\vec{B})$ which returns a noisy reward $r(\vec{B})$.

A standard UCB-type algorithm maintains empirical statistics for each arm and computes an index that serves as an upper confidence bound on the arm's true reward. For each arm $\vec{B}$ and time t, its upper-confidence-level index is given by

$$I_t(\vec{B}) := \hat{\mu}_t(\vec{B}) + f\big(N_t(\vec{B}), t\big),$$

where $N_t(\vec{B})$ is the number of times arm $\vec{B}$ has been selected and $\hat{\mu}_t(\vec{B})$ the empirical mean reward from $\vec{B}$. f is chosen so that, with high probability, $I_t(\vec{B})$ is an upper bound on the true mean reward $\mu(\vec{B})$. For example, the classical UCB1 algorithm sets

$$f\big(N_t(\vec{B}), t\big) = c\sqrt{\frac{\log t}{N_t(\vec{B})}}, \text{ for } c > 0.$$

Addressing the Combinatorial Explosion with StemArm Naively applying UCB to our setting would result in combinatorial explosion, so we incorporate the hierarchical tree structure from Branch And Bound to speed up our algorithms. A StemArm is a special arm that represents a *group* of candidate budget allocations. Instead of tracking every $\vec{B}$, we use StemArm to encapsulate less-promising budget allocations, which are grouped in polytope regions $\mathcal{B}_m \subseteq \mathcal{B}_0$. Formally,

Definition 3.9 (StemArm)**.** A StemArm represents a union of regions

$$\text{StemArm} = \cup_{m=1}^M \mathcal{B}_m.$$

When the StemArm is pulled, it splits out a most promising daughter arm:

$$\vec{B}_{\text{new}} := \underset{\vec{B} \in \text{StemArm}}{\arg\max}\ \texttt{LP}(\vec{B}),$$

And updates itself by partitioning the subregion that contains $\vec{B}_{\text{new}}$ to exclude it from the StemArm:

Let $\mathcal{B}^\star :=$ the subregion containing $\vec{B}_{\text{new}}$.

Split $\mathcal{B}^\star = \{\vec{B}_{\text{new}}\} \cup \mathcal{B}_{\text{new}_1} \cup \mathcal{B}_{\text{new}_2}$,

replace $\mathcal{B}^\star$ with $\mathcal{B}_{\text{new}_1}, \mathcal{B}_{\text{new}_2}$

Notice that all arms $\vec{B} \in$ StemArm (before they are split out) are never pulled, so they have no empirical history for UCB value. Instead the StemArm's UCB index is assigned as the upperbound of all its arms $\max_{\vec{B} \in \text{StemArm}} \texttt{LP}(\vec{B})$. For convenience, denote it as $\texttt{LP}(\text{StemArm})$. Note that because every pull of the StemArm splits out the daughter arm with the highest LP value, the StemArm's UCB index $\texttt{LP}(\text{StemArm})$ decreases during the Algorithm when it splits everytime.

Putting the MAB framework and StemArm together, the Mitosis algorithm operates as follows. We begin the MAB with the candidate arms containing only a StemArm, representing the entire feasible region $\mathcal{B}_0$. At each round, the algorithm selects from candidate arms (either a standard arm $\vec{B}$ or a StemArm) with the highest UCB index. When a standard arm is pulled, we run $r_t(\vec{B}) \leftarrow \text{Oracle}_{\text{small}}(\vec{B})$ to update its empirical statistics. When a StemArm is selected, it splits out a new arm into candidate arms and we pull it. We present pseudocode in Algorithm 1. The Mitosis algorithm is named for how the StemArm 'buds' new arms progressively during the algorithm, similar to cell division in mitosis.

Finally, we establish that our approach retains the no-regret guarantees of classical MAB algorithms:

Theorem 3. [No-Regret of the Mitosis Algorithm] Let $\mathcal{A}$ denote the set of arms that have been pulled. After running the algorithm for T rounds, the cumulative regret

$$R(T) \triangleq \sum_{t=1}^{T} (\mu^\star - \mu_t)$$

satisfies

$$R(T) = \sum_{\vec{B}\in\mathcal{A}} \mathbb{E}[N_T(\vec{B})]\,\Delta(\vec{B}) = O\left(\sum_{\vec{B}\in\mathcal{A}} \frac{\log T}{\Delta(\vec{B})}\right),$$

which matches the UCB1 regret bound.

To sum up, the the Mitosis marries the no-regret MAB algorithms with the StemArm structure. We leverage a faster but noisy oracle $\text{Oracle}_{\text{small}}$ and use a no-regret MAB algorithm to guide the explore. The combinatorial explosion in the number of arms is addressed by grouping less-promising arms into StemArm.

4 Experiments

We evaluate our proposed policies on two types of data: synthetic and real. In the following sections, we describe the experimental setup and report results separately for each case.

4.1 Experiments on Synthetic Data

Setup We formulate the food rescue volunteer notification problem as an instance of the Contextual Budget Bandit. In this setting, there are K regions (food-donation sources) and N volunteers who can be notified to pick up a donation. Figure 1 Provides an illustrative example. At each epoch $t \le T$, a trip arises from some region $k \in [K]$, and the decision is to notify a set of volunteers via actions a_i^t, taking into account their current states s_i^t and the region context.

The states of volunteers are governed by the following transition dynamics:

- Only active volunteers ($s_i = 1$) can pick up tasks ($r_i(0, a_i; k) = 0$ for all i, a_i, k).
- A notified volunteer ($a_i = 1$) is more likely to pick up a task, i.e., $r_i(1, 1; k) \ge r_i(1, 0; k)$ for all i, k.
- A notified volunteer is more likely to become inactive:

$$P_i[s_i^{t+1} = 0 \mid s_i, a_i = 1] \ge P_i[s_i^{t+1} = 0 \mid s_i, a_i = 0].$$

The decision maker's reward is defined as the total expected pick-up rate over all volunteers.

Volunteer Activity We describe the construction of two synthetic setups that capture different volunteer dynamics in the food rescue problem (details in Appendix D.1)

Algorithm 1 Mitosis

Input: Feasible region $\mathcal{B}_0$, LP upper-bound function `LP`, fast oracle $\text{Oracle}_{\text{small}}$
Auxiliary: UCB-type no-regret algorithm $I_t(\cdot)$
Output: Budget allocation $\vec{B}^*$ with high empirical reward

Initialize:
- Initialize StemArm := $\{\mathcal{B}_0\}$.
- Candidate set (heap) $\mathcal{A} \leftarrow \{\text{StemArm}\}$.
- Set time $t \leftarrow 0$.

while $t < T$ **and** stopping condition not met **do**
  Select from candidate set w.r.t. UCB index:
$$a^* \leftarrow \arg\max_{a\in\mathcal{A}} I_t(a),$$
  if a^* is a **standard arm** (i.e., corresponds to a specific $\vec{B}$) **then**
    Pull arm and observe reward $r_t(a^*) \leftarrow \text{Oracle}_{\text{small}}(\vec{B})$.
    Update the empirical statistics $N_t(a^*)$ and $\hat{\mu}_t(a^*)$.
  else
    // a^* **is a StemArm; splits out and pull new arm.**
    StemArm splits arm a^* to obtain a new standard arm a' with allocation $\vec{B}_{a'} = \vec{B}_{\text{new}}$.
    Pull new arm a'.
    Insert a' into the candidate set: $\mathcal{A} \leftarrow \mathcal{A} \cup \{a'\}$.
  end if
  $t \leftarrow t + 1$, update the candidate set $\mathcal{A}$ with $I_{t+1}(\cdot)$.
end while
return The allocation $\vec{B}^*$ corresponding to the arm in $\mathcal{A}$ with the highest empirical mean reward.

- **High Activeness** The N volunteers and K regions are randomly distributed over a two-dimensional plane. Each volunteer's decision to pick up a task is influenced by factors such as region popularity, distance, and personal engagement history.
- **Low Activeness** We introduce more challenging conditions. by partitioning the K a subset of *nasty* regions and its complement. In nasty regions, volunteers experience high pick-up rates that tend to deactivate them quickly, whereas in the remaining regions the rates are significantly lower.

The Effect of Volunteer Abundance and Budget on COcc's Performance Activity reflects the scarcity of volunteer resources. We can systematically vary the abundance of volunteers by tuning an *abundance ratio* $\rho_{\text{Abundance}} \in [0, 1]$: $\rho_{\text{Abundance}}$ of the N volunteers follow the High Activeness dynamics, while the remainder operate under the Low Activeness dynamics.

We systematically vary Abundance $\rho_{\text{Abundance}}$ and budget B to compare the performance between COcc against the optimal Mitosis Algorithm, and plot the heatmap of COcc's performance in Figure 2. When the volunteer resource is scarce

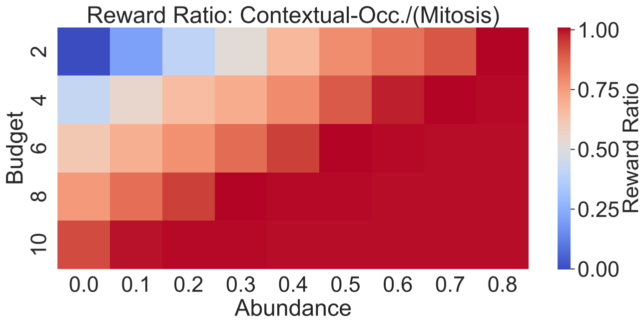

Figure 2: Reward ratio of COcc vs. Mitosis for [20] arms and [3] contexts across 32 seeds, varying Budget (vertical) and Abundance Ratio (horizontal). Red indicates near-optimal performance; blue indicates underperformance.

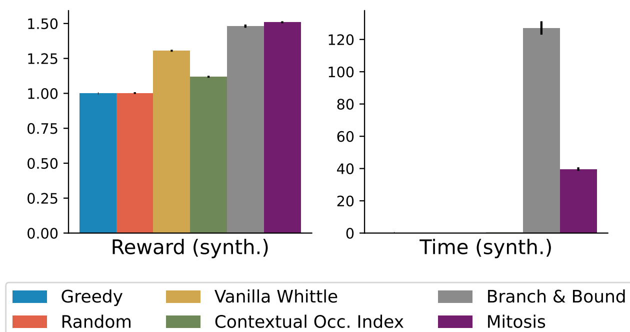

Figure 3: Main synthetic experiment with 50 arms, 3 contexts, 0.1 budget proportion, and 50% abundance. Bars show mean reward (left) and runtime (right) over 32 seeds. Mitosis yields the highest reward at significantly lower computation than Branch And Bound.

(low $\rho_{\text{Abundance}}$) *or* the notification budget B is small, COcc tends to lag behind compared to Mitosis (see top-left blue cells of Figure 2, where the reward ratio is almost close to 0). Low budget and low $\rho_{\text{Abundance}}$ show joint degrade effect for COcc.

Main Experiment For food rescue problem formulated as CBB using synthetic data, we compare our proposed policies (CBB and Mitosis) with several benchmarks: *Random* policy that selects arms uniformly at random; *Greedy* policy that selects arms with the highest immediate reward $r_i(s_i^t, 1, k)$, the aforementioned Vanilla Whittle policy from standard RMAB, and Branch And Bound. We report cumulative reward normalized by Random and runtime, measured in seconds.

In a representative synthetic instance (50 arms, 3 contexts, budget$=5$, 50% abundance), we run main experiments for the six aforementioned policies across 32 seeds (100 trials each). As shown in Figure 3 Mitosis (purple) achieves the highest reward overall, while Branch And Bound (gray) is also strong but much slower. The simpler baselines (Greedy, Random, Vanilla Whittle, and COcc), though taking almost no time to run, provide moderate to lower rewards, with the COcc notably underperforming at this abundance level.

Ablation Studies on synthetic data are conducted using 32 seeds by varying the number of volunteers ($N = 100, 200$), the number of regions ($K = 5, 10$) and budgets ($B = 2, 10$) respectively. Figure 4 shows the rewards of the various algorithms when varying N and B. Due to page limit, we defer the time plots and other reward plots to Appendix D.2. The Mitosis and Branch And Bound policies consistently perform best. COcc's performance, as the scale of the problem increases in either N, K, B, catches up with optimal, indicating that COcc performs generally better in larger scale problems.

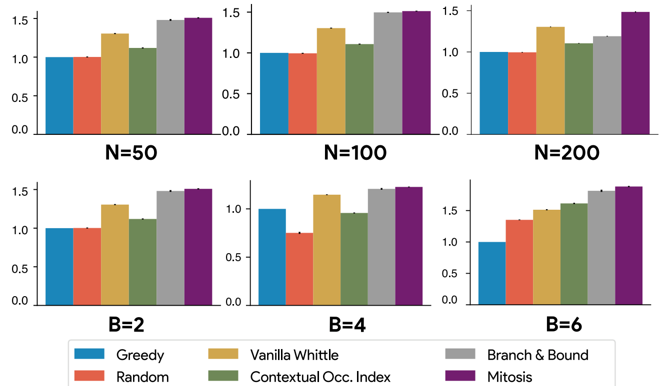

Figure 4: Ablations on Synthetic Data: Changing # Volunteers (First Row) and # budget (Second Row).

Overall, in synthetic food rescue CBB problem, Mitosis provides optimal reward comparative to Branch And Bound with modest runtime. COcc—though much faster—does not fully match Mitosis's optimal performance, and catches up as the problem scales up.

4.2 Experiments on Real Data

The experimental framework for real data mirrors that of the synthetic instance. The key difference is that the volunteers' and regions' attributes (historical engagement, location, and other idiosyncratic factors) are obtained from real-world datasets. We sample from a total pool of more than 500 thousand volunteers to construct the CBB instance. The state transitions and reward definitions remain identical to those described for synthetic data, ensuring that the same policies can be fairly compared across both domains.

We evaluate the same set of policies as in the synthetic experiments: COcc, Mitosis, Random, Greedy, Vanilla Whittle, and Branch And Bound. The main experiment on a Real Instance is run using 32 seeds with 100 trials per seed (Figure 5 and performance is measured in terms of the normalized reward (total accumulated reward divided by that of the random policy).

On real data, the context-aware methods (COcc, Branch And Bound and Mitosis) outperform Greedy, Random and Vanilla Whittle. Branch And Bound yields the highest average reward but requires disproportionately longer runtimes. By contrast, Mitosis nearly matches Branch And Bound while significantly reducing computation. Notably, COcc catches up with Mitosis—confirming it benefits from real-world attribute structure. The policies' performance trend is sim-

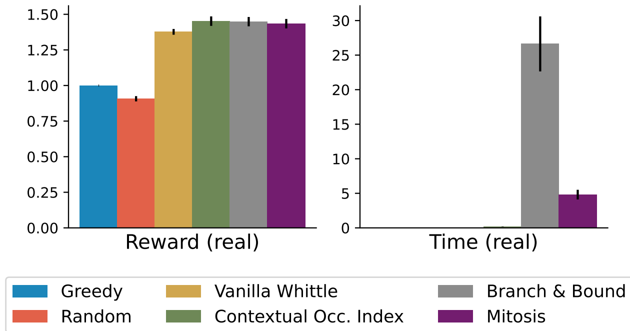

Figure 5: Experiment on real data. Bars show mean reward (left) and runtime (right) across 32 seeds (hatched to distinct real food rescue data). COcc, Branch And Bound and Mitosis perform equally well. Branch And Bound takes significantly more time to reach optimal.

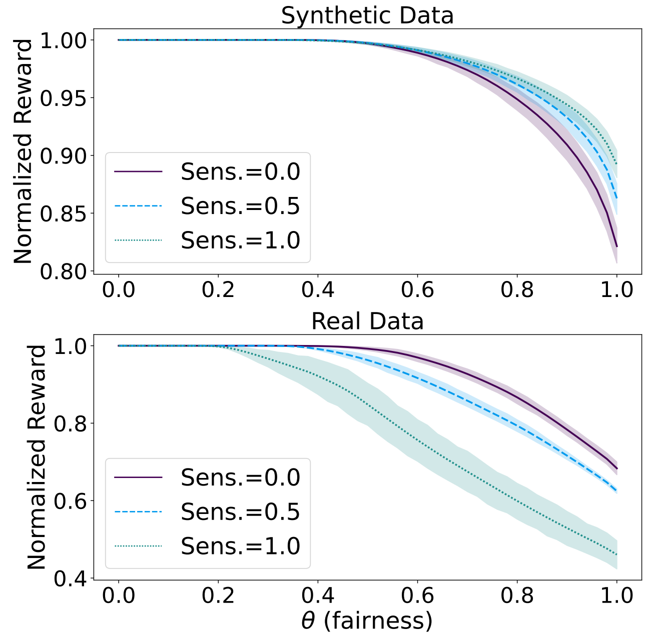

Figure 6: Pareto frontiers (θ vs. reward). Volunteer distance-sensitivity is shown in different line style. Heightened sensitivity makes fair solutions more costly in terms of total reward in real data.

ilar when we vary the number of volunteers, the number of regions and budget level in the ablation studies (see Appendix D.2 for details). This implies in application, COcc is sufficient for near-optimal performance. Mitosis guarantees optimality and is significantly faster than Branch And Bound.

5 Case Study: Fairness in Food Rescue

Geographical disparity in Food Rescue Food rescue organizations potentially suffer from geographic disparity where harder-to-reach areas are ignored. Over time, this can turn into a form of algorithmic discrimination, where certain regions or demographics are consistently underserved. We build on prior work that analyzes proportional fairness in RMABs [Li and Varakantham, 2022a; Wang *et al.*, 2024; Li and Varakantham, 2022b; Killian *et al.*, 2023] and develop a natural proportional fairness definition to enforce fairness across contexts in CBB. **The intuition is that a region's reward should be at least a fraction θ of the total reward multiplied by the region's occurrence probability.**

Definition 5.1 (θ-Fair)**.** For a contextual-RMAB instance, a budget allocation $\vec{B}$ is *θ-Fair* if the reward for each context divided by the occurrence probability of the context, is at least a fraction θ of its scaled share of the total reward in the solution of occupancy-measure LP (1):

$$\theta \underbrace{\left(\sum_i \sum_k \sum_{s_i, a_i} \mu_i(s_i, a_i; k) r_i(s_i, a_i; k) \right)}_{\text{total reward}} \quad (13)$$

$$\leq \underbrace{\frac{1}{f_k} \sum_i \sum_{s_i, a_i} \mu_i(s_i, a_i; k) r_i(s_i, a_i; k)}_{\text{reward for type } k}, \quad \forall k. \quad (14)$$

θ ranges on $[0, 1]$ and it tunes the fairness level. When $\theta = 0$ no fairness is imposed, while $\theta = 1$ enforces full fairness. For a fixed θ, the θ-Fair definition is linear in $\mu(\cdot, \cdot; \cdot)$. Hence, we can incorporate it into occupancy-measure LP and solve for a budget allocation $\vec{B}$ satisfying the θ-constraint, using estimates from the COcc.

Insights from Real Data Experiment We study how increasing fairness constraints ($\theta \in [0, 1]$) affect total reward, plotting this trade-off as a Pareto frontier in Figure 6 for both synthetic (left) and real (right) instances. A critical parameter in our model is volunteers' *sensitivity to distance*, which reduces pick-up rates for more remote locations. When sensitivity increases, it becomes harder to serve distant or underserved areas. As the fairness parameter θ grows under higher distance sensitivity, total reward declines more sharply.

6 Related Works

Restless Multi-Armed Bandits is a model of decision-making which extends multi-armed bandits so each arm has a state. While finding optimal actions for an RMAB is an NP-hard problem [Papadimitriou and Tsitsiklis, 1994], early work in the RMAB space proposed the Whittle index policy [Whittle, 1988] and demonstrated the asymptotic optimality of such a policy [Weber and Weiss, 1990a]. RMABs have seen increased attention recently due to its applicability for a range of real-world problems from maternal health [Mate *et al.*, 2022] to food rescue [Raman *et al.*, 2024] to autonomous vehicles [Li *et al.*, 2021]. Within contextual RMABs, lines of work include Bayesian transitions [Liang *et al.*, 2024], global contexts for demand modeling [Chen *et al.*, 2024], and using neural networks to predict indices [Guo and Wang, 2024]. Our work can be seen as an intersection between the application and technical lines of work, as we extend RMABs to varying budget contextual settings and apply these ideas to the food rescue domain.

Food Rescue are volunteer-driven organizations focusing on redistributing food Shi *et al.* [2020]. Prior works frequently utilize AI to model volunteer engagements [Manshadi and Rodilitz, 2020; Raman *et al.*, 2024], predicted trip difficulties [Shi *et al.*, 2024], and developed a recommender system for matching volunteers [Shi *et al.*, 2021]. We build on the volunteer engagement literature and address the issue of geographic disparity reported by prior work Shi *et al.* [2021].

Fairness is an increasingly important consideration in both business and non-profit organizations Bertsimas *et al.* [2012]; Liu and Garg [2024]. For RMAB, fairness is typically imposed on individual arms: Wang *et al.* [2024] define fairness as requiring a minimum long-term activation fraction for each arm; Li and Varakantham [2022b] propose a *soft fairness constraint*, or by setting an upperbound on the number of decision epochs since an arm was last activated (Li and Varakantham [2022a]). Fairness can also be defined over groups of arms. Killian *et al.* [2023] study minimax and max-Nash welfare objectives by imposing fairness on groups of arms, and Verma *et al.* [2024] enforce fairness with respect to the reward outcomes across groups. To the best of our knowledge, although RMABs with **contextual information** have been previously studied, our work is the first to consider fairness with respect to **context**.

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A Proof of Theorem 1

Theorem 1. For a CBB, denote the Whittle Index Policy's reward as $\mathcal{R}^{\text{VanillaWhittle}}$, and the optimal policy that satisfies context-specific budget constraint as $\mathcal{R}^{\text{ContextOpt}}$. There exists an instance where,

$$\frac{\mathcal{R}^{\text{ContextOpt}}}{\mathcal{R}^{\text{VanillaWhittle}}} \to \infty, \qquad \text{as } N \to \infty.$$

Proof. Consider a CBB instance with N stochastically identical arms. For simplicity we assume that the transition probabilities are such that each arm is always active ($s_i = 1$). Let there be two contexts, where context 1 occurs with probability $f_1 = 1 - \frac{1}{N}$ and context 2 occurs with probability $f_2 = \frac{1}{N}$. For each arm i, context 1 generates reward $r_i(s_i = 1, a_i = 1) = \frac{1}{N}$, context 2 generates reward $r_i(s_i = 1, a_i = 1) = N$. Suppose budget $B = 1$.

Consider the policy that leaves all arms idle at context 1, and pulls all N arms at context 2. The policy is feasible because its budget constraint $\vec{B} = (0, N)$ satisfies $f_1 \times 0 + f_2 \times N = \frac{1}{N} \times N = 1 = B$. Its average reward is N.

For Vanilla Whittle Index Policy, the good context 2 that has the high reward only occurs with probability $\frac{1}{N}$. And when it happens, we can only pull and get reward from *one* arm. So its average reward is $(1 - \frac{1}{N})\frac{1}{N} + \frac{1}{N} \times N = O(1)$. As $N \to \infty$, the gap between the two policies goes to infinity. □

B Dual of the occupancy-measure LP

Proof. Let $V_i(s_i, k), \forall i, s_i, k$ be the Lagrangian for constraint (2, 3). Let $\nu_i, \forall i$ be the Lagrangian multiplier for (4), and ρ for (5). The dual of the occupancy-measure LP (1) is

$$\min_{V,\rho,\nu} \sum_{i\in[N]} \nu_i + \rho B \tag{15}$$

$$s.t.\ V_i(s_i, k) + \nu_i \geq r_i(s_i, a_i; k) - \rho\mathbb{I}\{a_i = 1\} \tag{16}$$

$$+ \sum_{s_i', k'} V_i(s_i', k') P[s_i \to s_i' | a_i, k], \forall s_i, a_i, k \tag{17}$$

$$\rho \geq 0 \tag{18}$$

Then it is to show that for every (s_i, k) combination, at least one inequality in (16) is tight. By complementary slackness, a pair of optimal primal-dual variables $(\mu^\star; V^\star, \nu^\star, \rho^\star)$ would satisfy $\mu_i^\star(s_i, a_i; k) > 0$ only if constraint (16) is tight. (Assume non-degeneracy that every state-action s_i, k is of positive occupancy measure) for any arm i and state-context pair (s_i, k), at least one action a_i needs to be chosen, i.e. $\exists a_i, \mu_i^\star(s_i, a_i, k) > 0$, which implies that

$$V_i^\star(s_i, k) + \nu_i^\star = r_i(s_i, a_i; k) - \rho^\star \mathbb{I}\{a_i = 1\} + \sum_{s_i', k'} V_i^\star(s_i', k') P[s_i \to s_i' | a_i, k].$$

Combined with the rest of $a_i \in \mathcal{A}$ and inequalities in 16, we have

$$V_i^\star(s_i, k) + \nu_i^\star = \max_{a_i \in \{0,1\}} (r_i(s_i, a_i; k) - \rho^\star \mathbb{I}\{a_i = 1\} + \sum_{s_i', k'} V_i^\star(s_i', k') P[s_i \to s_i' | a_i, k]).$$

□

C Proof of Theorem 2

Theorem 2. The COcc's asymptotic approximation ratio compared to $\mathcal{R}^{\text{ContextOpt}}$ is bounded above by $\frac{5}{6}$.

Proof. **Outline** First, we formally establish the asymptotic framework, which is where the Whittle Index Policy for standard RMAB achieves optimality. Then we introduce how to analyze CBB's in this asymptotic regime. Finally, we present the instance where the $\frac{5}{6}$ bound is achieved.

Asymptotic Notion

We define the asymptotic notion for analyzing (sub-)optimality for CBB. It is same to the approach for standard RMAB, originally proposed by Weber and Weiss [1990b] for RMAB with stochastically identical arms and generalized to heterogeneous arms by Xiong *et al.* [2022]:

Definition C.1 (ρ-scaled CBB)**.** Fix a *Base* CBB instance with M arms

$$\langle M, \mathcal{S}, \mathcal{A}, K, \{r_i^k\}_{i\in[M],k\in\mathcal{K}}, \{P_i^k\}_{i\in[M],k\in\mathcal{K}}, \mathcal{F}\rangle.$$

With budget $B \in \mathbb{N}$.

Now, consider each arm being replicated ρ times, with the budget scaled by ρ as well. The new CBB instance has $\rho \times M$ arms, with each of the M arms in the base CBB repeated ρ times. Budget is scaled to ρB.

For a base contextual RMAB instance scaled with ρ, when there is no confusion about the base instance we're referring to, denote its reward for any policy π as

$$\mathcal{R}^\pi(\rho) := \lim_{T\to\infty} \mathbb{E}_{\vec{a}\sim\pi(\cdot),k\sim\mathcal{F}}\Big[\frac{1}{T}\sum_{t=1}^{T}\sum_{i\in[\rho M]} r_i^k(s_i^t, a_i^t)\Big].$$

For ρ-scaled CBB, notice that its reward upperbound from solving the occupancy-measure LP (1) simply scales with ρ:

$$\overline{\mathcal{R}}(\rho) = \rho\overline{\mathcal{R}}(1).$$

We refer to every arm $i \in [M]$ in the base instance as a **type-**i arm, and its ρ replicates in the ρ-scaled CBB as the ρ type-i arms.

Asymptotic System Behavior for CBB

In the following section we introduce a new method for analyzing the asymptotic behavior of CBB as $\rho \to \infty$. It is different from the standard approach of Weber and Weiss [1990b].

To ease the complication of notations, we describe our method with CBB that $r_i(s_i = 0, a = 0; k) = r_i(s_i = 0, a = 1; k) = 0, \forall i, k$, and transition probabilities $P_i^k s_i^{t+1} \mid s_i^t, a_i^t = 1] = P_i^k s_i^{t+1} \mid s_i^t, a_i^t = 0], \forall s_i^{t+1}, s_i^t$. In this way it is meaningless to pull inactive arms, since it makes no difference in rewards nor transition probabilities. Generalization to general CBB is without loss of generality.

We care about the proportion of active arms as $\rho \to \infty$ of the ρ. The following technical lemma characterizes the dynamic of arms:

Lemma 1. Denote as a_i^t the proportion of type-i active arms at any time point t under given policy π. Conditional on a_i^t and context k, a_i^{t+1}'s distribution converges to a Direc Delta function $\delta_{\text{shift}}(\cdot)$, shifted with $\mathbb{E}[a_i^{t+1} \mid a_i^t, k]$ as the total number of arms $\rho \to \infty$. In other words,

$$f(a_i^{t+1} \mid a_i^t, k) = \delta_{\mathbb{E}[a_i^{t+1}|a_i^t,k]}(a_i^{t+1}), \tag{19}$$

where, let $B_{i,k}$ be the number of active type-i arms pulled by the policy at context k:

$$\mathbb{E}[a_i^{t+1} \mid a_i^t, k] = \min(a_i^t, \frac{B_{i,k}}{\rho}) P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1] \tag{20}$$

$$+ (a_i^t - \min(a_i^t, \frac{B_{i,k}}{\rho}) \tag{21}$$

$$\times P_i^k s_i^{t+1} = 0 \mid s_i^t = 1, a_i^t = 1] \tag{22}$$

$$+ (1 - a_i^t) P_i^k s_i^{t+1} = 1 \mid s_i^t = 0] \tag{23}$$

Proof. The sketch of the proof is that, the **number** of active arms is sum of Binomial random variables, with parameters given the by the policy and transition probabilities. As total number of arms $\rho \to \infty$, each Binomial variable divided by ρ converges to (shifted) Direc Delta. Therefore, the *proportion* of active arms is also (shifted) Direc Delta.

Notes on Binomial Distribution To make later analysis clear, first consider a single binomial random variable X with N experiments and success rate p (i.e. $X \sim \text{Bin}(N, p)$). For any $x \in [0, 1]$ (assume Nx is integer):

$$P[X = Nx] = \binom{N}{Nx} p^{Nx} (1-p)^{N(1-x)}$$

$$\text{Apply Stirling's Formula: } n! \sim \sqrt{2\pi n} \cdot (\frac{n}{e})^n$$

$$= \sqrt{\frac{1}{x(1-x)N}} \cdot \left((\frac{p}{x})^x (\frac{1-p}{1-x})^{(1-x)} \right)^N$$

It can be verified that $(\frac{p}{x})^x (\frac{1-p}{1-x})^{(1-x)} < 1$ for $x \neq p$. Therefore, as $N \to \infty$

$$P[X = xN] = \begin{cases} \sqrt{\frac{1}{x(1-x)N}} \to \infty & x = p \\ \sqrt{\frac{1}{x(1-x)N}} \\ \times \mathcal{O}(\left((\frac{p}{x})^x (\frac{1-p}{1-x})^{(1-x)} \right)^N) \to 0 & x \neq p \end{cases}$$

Therefore, say if we let $f(x) = P[X = xN]$, $f(\cdot)$ is a shifted-to-p Direc Delta function.

Stationary Distribution Contextual Budget Bandit Let $A_i^t := a_i^t \rho$ denote the **number of active type-i arms** at time point t. Conditional on current A_i^t and context k,

- $\min(A_i^t, B_{i,k})$ arms are pulled, where each arm remains active w.p. $P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1]$.
- Each of $\rho - A_i^t$ inactive arms transfers back to active w.p. $P_i^k s_i^{t+1} = 1 \mid s_i^t = 0]$,

Therefore, the number of active arms at next period A_i^{t+1} is the sum of three binomial random variables:

$$\{A_i^{t+1} \mid A_i^t, k\} \tag{24}$$

$$\sim \underbrace{\text{Bin}(\min\{A_i^t, B_{i,k}\}, P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1])}_{\text{active arms pulled staying active}} \tag{25}$$

$$+ \underbrace{\text{Bin}(A_i^t - \min\{A_i^t, B_{i,k}\}, P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 0])}_{\text{idle active arms staying active}} \tag{26}$$

$$+ \underbrace{\text{Bin}(\rho - A_i^t, P_i^k s_i^{t+1} = 1 \mid s_i^t = 0])}_{\text{inactive arms transfer back to active}}. \tag{27}$$

Scaled by $\rho \to \infty$, each of the above binomial distribution converges to a Direc Delta function centered on its mean. Since adding up random variables is equivalent to taking convolution of their probability mass functions—Direc Delta functions are closed under convolution—random variable $a_i^{t+1} = \frac{A_i^{t+1}}{\rho}$'s probability mass function is a Direc Delta shifted by $\frac{1}{\rho}\mathbb{E}[A_i^{t+1} \mid A_i^t, k]$. $\square$

The lemma implies, the *proportion* of active arms a_i^t evolve "almost deterministically"—more precisely speaking, fix any policy π, if at current time step the proportion of active arms is a_i^t, context is k, the next time step will have $(\mathbb{E}[a_i^{t+1} \mid a_i^t, k])\%$ active arms almost surely, where $(\mathbb{E}[a_i^{t+1} \mid a_i^t, k])$ is given by the following:

$$\begin{aligned}
&\mathbb{E}[a_i^{t+1} \mid a_i^t, k] \\
=& \frac{1}{\rho}\mathbb{E}[A_i^{t+1} \mid A_i^t, k] \\
=& \frac{1}{\rho}\mathbb{E}[\underbrace{\text{Bin}(\min\{A_i^t, B_{i,k}\}, P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1])}_{\text{active arms pulled}} \\
&+ \underbrace{\text{Bin}(A_i^t - \min\{A_i^t, B_{i,k}\}, P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 0])}_{\text{untouched active arms}} \\
&+ \underbrace{\text{Bin}(\rho - A_i^t, q_i)}_{\text{inactive arms}}] \\
=& \frac{1}{\rho}(\min\{A_i^t, B_{i,k}\} \cdot P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1] \\
&+ (A_i^t - \min\{A_i^t, B_{i,k}\}) \cdot P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 0] \\
&+ (\rho - A_i^t) q_i)
\end{aligned}$$

Denote $\beta_{i,k} := \frac{B_{i,k}}{\rho}$:

$$\mathbb{E}[a_i^{t+1} \mid a_i^t, k] \tag{28}$$

$$= \min(a_i^t, \beta_{i,k}) \cdot P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 1] \tag{29}$$

$$+ \max(a_i^t - \beta_{i,k}, 0) \cdot P_i^k s_i^{t+1} = 1 \mid s_i^t = 1, a_i^t = 0] \tag{30}$$

$$+ (1 - a_i^t) P_i^k s_i^{t+1} = 1 \mid s_i^t = 0] \tag{31}$$

If, current time step's proportion of active arms is $x \in [0, 1]$, with probability f_k context k occurs, then the next time step's active-arm proportion will be $y = \mathbb{E}[a_i^{t+1} \mid x, k]$ (as given in 29-31) w.p. f_k. And for each y, define its inverse

$$\mathcal{X}(y) := \{(x,k) : \mathbb{E}[a_i^{t+1} \mid x, k] = y\}. \tag{32}$$

Denote the stationary distribution of proportion of active arms as $\pi : [0,1] \to [0,1]$, it should satisfy:

$$\pi(y) = \sum_{(x,k)\in\mathcal{X}(y)} f_k \pi(x). \tag{33}$$

C.1 A 5/6 Approximation Upperbound.

An adversarial instance Consider a base CBB example with only one type of arm (i.e., $M = 1$). Let there be ρ copies of this arm in the scaled setting as $\rho \to \infty$. We drop the index i for convenience. Suppose there are two contexts, $k \in \{1, 2\}$, each occurring with probability $f_1 = f_2 = 0.5$.

Let $\epsilon > 0$. The transition probabilities and rewards are defined as follows.

- Context 1: transition probabilities is

$$\begin{aligned}
&P^1[s^{t+1} = 1 \mid s = 1, a = 1] = 1 - \epsilon \\
&P^1[s^{t+1} = 0 \mid s = 1, a = 1] = \epsilon \\
&P^1[s^{t+1} = 1 \mid s = 1, a = 0] = 1 \\
&P^1[s^{t+1} = 0 \mid s = 1, a = 0] = 0 \\
&P^1[s^{t+1} = 1 \mid s = 1, \forall a = 0, 1] = 1 \\
&P^1[s^{t+1} = 0 \mid s = 1, \forall a = 0, 1] = 0
\end{aligned}$$

  reward for context 1:

$$\begin{aligned}
&r(s^t = 1, a^t = 1; k = 1) = 1 \\
&r(s^t = 1, a^t = 0; k = 1) = 0 \\
&r(s^t = 0, a^t = 1; k = 1) = 0 \\
&r(s^t = 1, a^t = 0; k = 1) = 0
\end{aligned}$$

- Context 2: transition probabilities is

$$\begin{aligned}
&P^2[s^{t+1} = 1 \mid s = 1, a = 1] = 0 \\
&P^2[s^{t+1} = 0 \mid s = 1, a = 1] = 1 \\
&P^2[s^{t+1} = 1 \mid s = 1, a = 0] = 1 \\
&P^2[s^{t+1} = 0] \mid s = 1, a = 0] = 0 \\
&P^2[s^{t+1} = 1 \mid s = 1, \forall a = 0, 1] = 1 \\
&P^2[s^{t+1} = 0 \mid s = 1, \forall a = 0, 1] = 0
\end{aligned}$$

  reward for context 2:

$$\begin{aligned}
&r(s^t = 1, a^t = 1; k = 1) = 1 + \epsilon \\
&r(s^t = 1, a^t = 0; k = 1) = 0 \\
&r(s^t = 0, a^t = 1; k = 1) = 0 \\
&r(s^t = 1, a^t = 0; k = 1) = 0
\end{aligned}$$

Bugdet Assume that budget is $1/3$ of the number of total arms. I.e. in the ρ-scaled instance, $B = \lfloor \frac{1}{3} \rfloor$. As the scaling factor $\rho \to \infty$, we can without loss of generality assumes that it's an interger.

The Reward for COcc The occupancy-measure LP for the base instance simplifies to

$$\begin{aligned}
\max_{\mu, B_k} \quad & \mu(1,1,1) + \mu(1,1,2)(1+\epsilon) \\
\text{subject to} \quad & \\
& (1 - P[s = 1]) = \epsilon\mu(1,1,1) + \mu(1,1,2) \\
& \mu(1,1,k) \le \frac{1}{2} P[s = 1], \forall k = 1, 2 \\
& \mu(1,1,1) + \mu(1,1,2) \le \frac{1}{3}
\end{aligned}$$

The COcc then allocate budget following the optimal solution ($\mu^\star$) of the occupancy-measure LP. For the ρ-scaled CBB with total budget $B = \frac{1}{3}\rho$, the budget allocation of COcc is

$$\begin{aligned}
B_1 &= \rho \times \frac{1}{f_1} \mu^\star(1,1,1) = 0, \\
B_2 &= \rho \times \frac{1}{f_2} \mu^\star(1,1,2) = \frac{2}{3}.
\end{aligned}$$

From (29-31) we obtain, as $\rho \to \infty$, the transition dynamic of the proportion of active arms $x^t \to x^{t+1}$ in RMAB:

- With probability $f_1 = 0.5$, context $k = 1$:
$$x^{t+1} = \mathbb{E}[a_i^{t+1} \mid x^t, k] = 1;$$
- With probability $f_2 = 0.5$, context $k = 2$:
$$x^{t+1} = \mathbb{E}[a_i^{t+1} \mid x^t, k] = \max(x^t - \frac{2}{3}, 0) + 1 - x^t.$$

From 32 and 33 we obtain the stationary distribution π under `Policy`*: (actually, guess-and-verify)

$$\begin{aligned}
&\pi(\frac{1}{3}) = \frac{1}{3}, \\
&\pi(\frac{2}{3}) = \frac{1}{6} \\
&\pi(1) = \frac{1}{2}, \\
&\pi(x) = 0, \text{otw.}
\end{aligned}$$

When the proportion of active arms$= \frac{1}{3}$—only half of the budget is utilized. This happens, as give above, w.p. $\pi(\frac{1}{3}) = \frac{1}{3}$. So the reward as $\rho \to \infty$ is

$$\begin{aligned}
&\mathcal{R}^{\text{COcc}}(\rho) \\
&= f_2(\frac{1}{3}\rho\pi(\frac{1}{3}) + \frac{2}{3}\rho(\pi(\frac{2}{3}) + \pi(1)) \\
&= 0.5\rho(\frac{1}{9} + \frac{4}{9}) = \frac{5}{18}\rho
\end{aligned}$$

Optimal Budget Allocation However, notice that the other context $k = 1$ is almost always active (it has probability $p = \epsilon$ of transfer to inactive). Therefore, if we allocate all budget to context 1:

$$B_1 = \frac{2}{3}, B_2 = 0$$

. The stationary reward for the optimal budget allocation is

$$\mathcal{R}^{\text{ContextOpt}}(\rho) = \frac{1}{3}\rho$$

Therefore, the COcc's approximation is bounded above by $\frac{5}{6}$.

$$\lim_{\rho\to\infty} \frac{\mathcal{R}^{\text{COcc}}(\rho)}{\mathcal{R}^{\text{ContextOpt}}(\rho)} = \frac{5}{6}.$$

□

C.2 Remark: Closed-form unavailable

Ending remark for this Appendix section, and as a complement to the asymptotic analysis of CBB, we provided the following example, where, the closed-form solution of the staionary distribution of the proportion of active arms can only be calculated numerically but not characterized in clean closed-form as the above example.

Single-Type Base Example 2 Consider a base CBB example with only one type of arm (i.e., $M = 1$). Let there be ρ copies of this arm in the scaled setting as $\rho \to \infty$. We drop the index i for convenience. Suppose there are two contexts, $k \in \{1, 2\}$, each occurring with probability $f_1 = f_2 = 0.5$. **Transition Probabilities and Rewards.** Let $\epsilon > 0$. The transition probabilities and rewards are defined as follows.

Transition Probabilities and Rewards. Let $\epsilon > 0$. The transition probabilities and rewards are defined as follows.

- **Context 1:** *Transition probabilities*:

$$\begin{aligned}
P^1[s^{t+1} &= 1 \mid s = 1, a = 1] = 1 - \epsilon,\\
P^1[s^{t+1} &= 0 \mid s = 1, a = 1] = \epsilon,\\
P^1[s^{t+1} &= 1 \mid s = 1, a = 0] = 1,\\
P^1[s^{t+1} &= 0 \mid s = 1, a = 0] = 0,\\
P^1[s^{t+1} &= 1 \mid s = 0, \forall a = 0, 1] = \frac{1}{2}\\
P^1[s^{t+1} &= 0 \mid s = 0, \forall a = 0, 1] = \frac{1}{2}
\end{aligned}$$

  Rewards:

$$\begin{aligned}
r(s^t &= 1, a^t = 1; k = 1) = 1,\\
r(s^t &= 1, a^t = 0; k = 1) = 0,\\
r(s^t &= 0, a^t = 1; k = 1) = 0,\\
r(s^t &= 0, a^t = 0; k = 1) = 0.
\end{aligned}$$

- **Context 2:** *Transition probabilities*:

$$\begin{aligned}
P^2[s^{t+1} &= 1 \mid s = 1, a = 1] = 0,\\
P^2[s^{t+1} &= 0 \mid s = 1, a = 1] = 1,\\
P^2[s^{t+1} &= 1 \mid s = 1, a = 0] = 1,\\
P^2[s^{t+1} &= 0 \mid s = 1, a = 0] = 0,\\
P^2[s^{t+1} &= 1 \mid s = 0, \forall a = 0, 1] = \frac{1}{2}\\
P^2[s^{t+1} &= 0 \mid s = 0, \forall a = 0, 1] = \frac{1}{2}
\end{aligned}$$

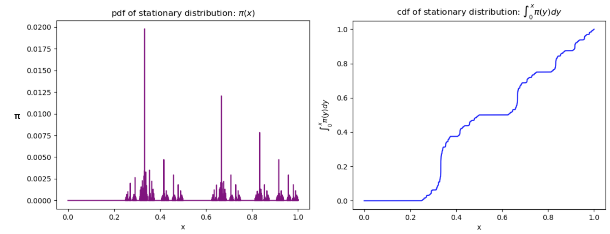

Figure 7: Calculated stationary distribution.

  Rewards:

$$\begin{aligned}
r(s^t &= 1, a^t = 1; k = 2) = 1 + \epsilon,\\
r(s^t &= 1, a^t = 0; k = 2) = 0,\\
r(s^t &= 0, a^t = 1; k = 2) = 0,\\
r(s^t &= 0, a^t = 0; k = 2) = 0.
\end{aligned}$$

Budget Constraint. Assume the budget in each round is a fraction of the total number of arms. For concreteness, let the budget be

$$B = \left\lfloor \tfrac{1}{4}\,\rho \right\rfloor,$$

so that we may activate at most $\lfloor \frac{\rho}{4} \rfloor$ arms (out of ρ). As $\rho \to \infty$, we can assume without loss of generality that $B = \frac{\rho}{3}$ is an integer.

The occupancy-measure LP simplifies to

$$\begin{aligned}
\max_{\mu,\,B_k}\quad & \mu(1,1,1) \;+\; \mu(1,1,2)\,\big(1+\epsilon\big)\\
\text{subject to}\quad & \tfrac{1}{2}\big(1 - P[s=1]\big) \;=\; \epsilon\,\mu(1,1,1) \;+\; \mu(1,1,2),\\
& \mu(1,1,k) \;\le\; \tfrac{1}{2}\,P[s=1], \quad \forall\, k \in \{1,2\},\\
& \mu(1,1,1) + \mu(1,1,2) \;\le\; 0.25
\end{aligned}$$

The optimal solution is $P^\star[s = 1] = 0.5, \mu^\star(1, 1, 1) = 0, \mu^\star(1, 1, 2) = 0.25$. Similarly, COcc would allocate budget so that

$$\begin{aligned}
B_1 &= 0,\\
B_2 &= 0.5\rho.
\end{aligned}$$

Therefore, from 32 and 33 we obtain for the stationary distribution π:

$$\pi(y) = \begin{cases} 0 & y \in (0, \frac{1}{4})\\ \frac{1}{2}\pi(\frac{1}{2}) & y = \frac{1}{4}\\ \frac{1}{2}\pi(1-2y) + \frac{1}{2}\pi(2y) & y \in (\frac{1}{4}, \frac{1}{2}]\\ \frac{1}{2}\pi(2y-1) & y \in (\frac{1}{2}, 1]. \end{cases} \tag{34}$$

It doesn't have a clean closed-form solution. But the stationary of proportion of active arms ($\pi(\cdot)$ can be solved numerically, as shown in Figure 7. As shown in Figure 7, for non-trivial probability, the proportion of active arms is less than 0.5—less than the required active arms to pull. The stationary reward can be calculated as

$$\begin{aligned}
&\mathcal{R}^{\text{COcc}} \\
&= \int_0^1 \sum_k f_k r(k) \rho \min(\beta_k, x) \pi(x)\, dx \\
&= \rho \int_0^1 \frac{1}{2}(1+\epsilon) \min(\frac{1}{2}, x) \pi(x)\, dx \\
&\approx \rho(1+\epsilon) 0.214.
\end{aligned}$$

However, notice that the other context $k = 1$ is almost always acive (it has probability $p = \epsilon$ of transfer to inactive). Therefore, if we allocate all budget to it—almost all arms will be active all the time, and reward of $r(1) = 1$ can be accured at every pull. By back-on-the-envelope calculation, under this budget allocation (all to context 1) the system generate (almost) exactly $\frac{1}{4}$ reward. Therefore, this instance give an lowerbound of $0.214/0.25 = 0.856$ impossibility lowerbound for the LP-induced budgets.

D Experiment Details: Design and Implementation

D.1 Low/High Activeness in Synthetic Food RescueCBB

We blend two types of synthetic setups to merge and simulate different dynamics in formulating the food rescue CBB:

High Activeness
In the High Activeness instance, N volunteers and K regions are randomly positioned on a two-dimensional plane. Each volunteer and region is associated with a location and attributes—namely, volunteer activeness, region popularity, and a historical record H_i (which, in turn, influences the context probabilities f_k). For every volunteer i and region k, we define the pick-up rate as

$$p_{i,k} = \exp\left(\alpha\, \text{pop}_k - \gamma\, d(i,k) + \beta\, \frac{|H_i|}{H_{\max}}\right),$$

where
- α is the parameter capturing the influence of region popularity (with pop_k denoting the popularity of region k), - γ is the distance sensitivity parameter (with $d(i,k)$ representing the distance between volunteer i and region k), - β is the parameter reflecting volunteer activeness (with $|H_i|$ being the size of volunteer i's history), and - $H_{\max}$ is a normalization constant.

Transition dynamics are such that an active volunteer (state $s = 1$) who is notified (action $a = 1$) picks up the task with probability p_{ik} and may then become inactive. The immediate reward for a notification is a function of region popularity and p_{ik}.

Low Activeness
In addition to the High Activeness instance, we define a Low Activeness instance to capture more challenging dynamics within the CBB framework. It is motivated by the scenario that induces the theoretical $5/6$ inefficiency for COcc in Theorem 2 (see Appendix C.1 for details).

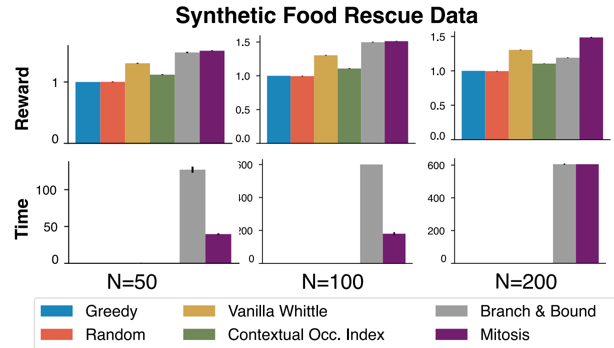

Figure 8: Ablation Experiments on Synthetic Food Rescue Experiments, Varying Number of Volunteers

The K regions are partitioned into *nasty* regions ($\mathcal{K}_{\text{nasty}} \subsetneq [K]$ and its complement. The nasty regions are adversarially designed such that, for any volunteer i, the pick-up probabilities p_{ik} for $k \in \mathcal{K}_{\text{nasty}}$ are drawn uniformly around a high mean (e.g., centered at 0.95), making these regions very attractive and yielding a high probability of transitioning a volunteer to an inactive state. In contrast, for regions $k \notin \mathcal{K}_{\text{nasty}}$ (the "cheap" regions), the transition probabilities are concentrated around a low mean (e.g., centered at 0.05). Recovery probabilities q_i for volunteers are generated around a prescribed mean (e.g., 0.2). Moreover, the reward structure is modified so that notifications in nasty regions yield a slightly elevated immediate reward (e.g., $1 + \varepsilon$) to reflect their allure despite the adverse long-term effect. This construction presents a challenge for COcc theoretically, as shown in the proof of Theorem 2 in its proof in Appendix C.1.

Blended Instance
Finally, we construct a *Blended Instance* that merges High Activeness and Low Activeness dynamics. A fraction $\rho_{\text{Abundace}} \in [0, 1]$ (termed the *sensitivity ratio*) of the N volunteers is designated to follow Low Activeness dynamics, while the remaining $N - N_{\text{active}}$ volunteers follow High Activeness dynamics. Formally, we set

$$N_{\text{active}} = \lfloor \rho_{\text{Abundace}} N \rfloor,$$

and generate two independent instances over the same set of K regions:

(i) A *high activity instance* with N_{active} volunteers. The transition dynamics and rewards are constructed as described in Section D.1

(ii) A *low activity instance* with $N - N_{\text{active}}$ volunteers, constructed as described in Section D.1.

D.2 Ablation Experiments on Synthetic Food Rescue Instance

Below, we summarize the ablation study results on synthetic data, where we systematically vary the number of volunteers (N), the number of regions (K), and the budget (B).

- **Varying number of volunteers for $N = 50, 100, 200$, fix $K = 3$ regions and budget be 5% number of volunteers (Figure 8):** As N increases, Mitosis (purple)

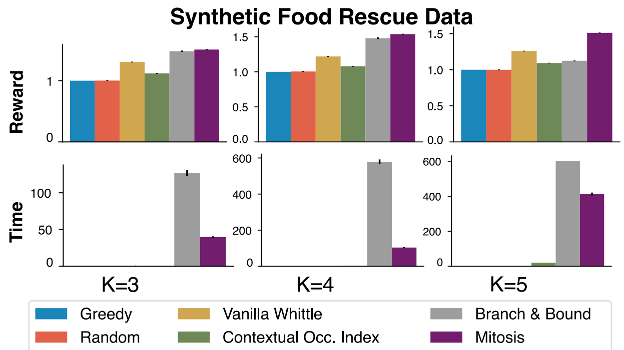

Figure 9: Ablation Experiments on Synthetic Food Rescue Experiments, Varying Number of Contexts

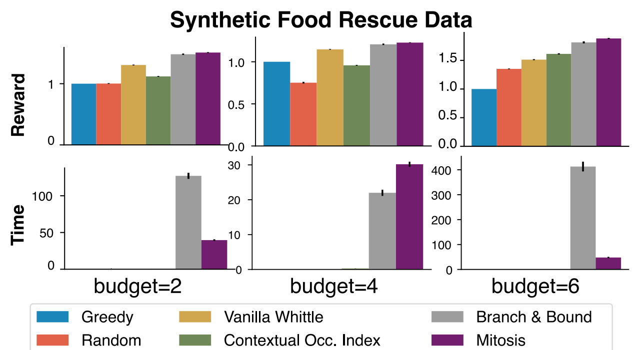

Figure 10: Ablation Experiments on Synthetic Food Rescue Experiments, Varying Budget

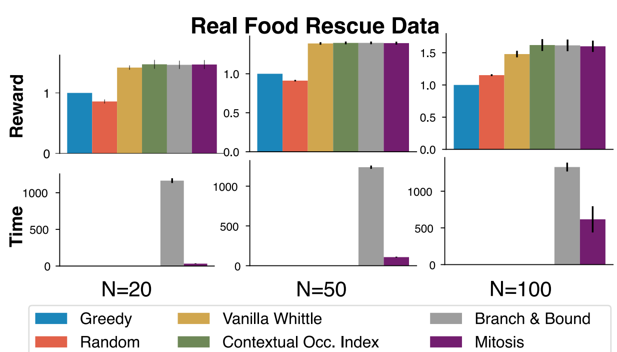

Figure 11: Ablation Experiments on Real Food Rescue Experiments, Varying Number of Volunteers

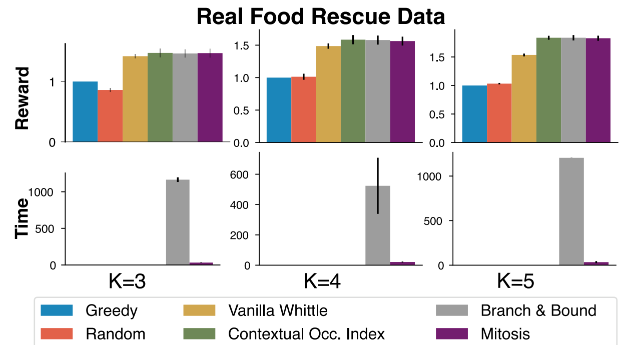

Figure 12: Ablation Experiments on Real Food Rescue Experiments, Varying Number of Contexts

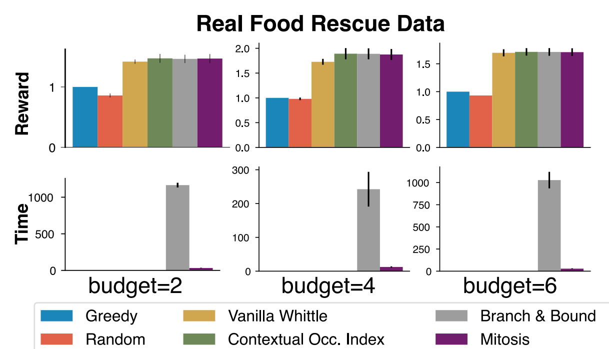

Figure 13: Ablation Experiments on Synthetic Food Rescue Experiments, Varying Budget

consistently leads in reward and remains much faster than Branch And Bound (gray). Note that in $N = 200$ both Branch And Bound and Mitosis reach the time limit (600s) and is terminated, but sill within the same time limit, the Mitosis's solution is more than that of Branch And Bound's, demonstrating that Mitosis is much faster. COcc (green) still lags in reward, indicating it does not fully exploit the increased volunteer pool.

- **Varying number of regions for $K = 3, 4, 5$, fix number of volunteers $N = 50$, budget $B = 5$ (Figure 9):** With more regions, the search space for budget increases exponentially. Branch And Bound maintains a slight reward edge but at a steep runtime cost, it times out already at $k = 4$. Mitosis remains best and is faster compared to Branch And Bound. COcc gains some benefit but continues to underperform compared to the optimal.
- **Varying budget $B = 2, 4, 6$, fix volunteers $N = 50$, regions $K = 3$ (Figure 10):** Increasing B allows more notifications, boosting Mitosis substantially while also helping COcc close some of the gap. Once again, Branch And Bound yields top-tier rewards but incurs much higher computation time.

D.3 Ablation Experiments on Real Food Rescue Data

Similar as synthetic data's ablations, we systematically vary the number of volunteers (N), the number of regions (K), and the budget (B) of CBB constructed on real food rescue data. Results are shown in

- Figure 11: changing $N = 20, 50, 100$ while maintain number of regions $K = 3$, budget $B = 2$.
- Figure 12: changing $K = 3, 4, 5$ while maintaining number of regions $N = 20$, budget $B = 2$.
- Figure 13: changing $B = 2, 4, 6$ while maintaining number of volunteers $N = 20$, number of regions $K = 3$.

As the scale of the instance increases (N, K or B increases), Vanilla Whittle performance grows worse compared to COcc, Branch And Bound, Mitosis which they perform similarly. This shows that (i) when the scale of the problem increase, it is necessary to introduce context-aware policies to reach optimal performance (ii) in application, COcc is sufficient for near-optimal performance. Mitosis guarantees optimality and is significantly faster than Branch And Bound.

E Generalizability to Other Application Areas

While we ground our methodological work in food rescue volunteer engagement, the Contextual Budget Bandit model and its algorithms are applicable to a variety of domains. In this section, we describe a few of these applications.

Digital agriculture Smallholder farmers in the global south feed their countries yet are vulnerable to climate change and market fluctuation. Agriculture chatbots are a promising direction to empower smallholder farmers, as evidenced, for example, in an NSF report [Guérin *et al.*, 2024]. In collaboration with Organization X, we have a chatbot which sends out regular nudges about farming practices to over 10,000 farmers in India, Kenya, and Nigeria. However, nudges of different topics have different conversion rates. Pest control tips during the pest season address an urgent problem, usually resulting in high conversion rates. Meanwhile, watering tips are preventive measures, which often have lower conversion rates by the farmers. Thus, when planning the engagement strategy over time, one would want to assign different nudging budgets to different topics of nudges, and model it as a CBB. Each farmer is an arm. At each time step, we have a nudge topic as context, and we decide on a budget of how many farmers to notify and the arm selection of who to notify. Rewards are determined based on farmer's engagement response.

Peer review In peer review, journals select reviewers for submissions where selection impacts future reviewer availability [Payan and Zick, 2021]. For a given paper, the goal is to select a subset of available reviewers with the relevant subject-matter expertise. However, submissions differ from one another. For example, submissions that are extra long, that involves heavy theoretical analysis, or that do not study the trendy topics might have lower chance of getting reviewers to agree to review. Thus, when the editor plans reviewing invitations over time, they would want to send different numbers of invitations to different kinds of submissions, and model it as a CBB. Each potential reviewer is an arm. At each time step, we have a submission type as context, and we decide on a budget of how many potential reviewers to reach out to, and the arm selection of who to reach out to. Rewards are determined based on the reviewers' response.

F Pseudocode for Branch And Bound Algorithm

Algorithm 2 Branch And Bound

Input: Feasible Region $\mathcal{B}_0$, $\texttt{LP}$, Oracle
Output: Budget Allocation $\vec{B}^*$

Initialize: $R^{\text{OPT}} \leftarrow -\infty$, $\vec{B}^{\text{OPT}} \leftarrow \texttt{None}$, Queue $Q \leftarrow \{\mathcal{B}_0\}$.
while $Q \neq \varnothing$ **do**
  Dequeue $\mathcal{B} \leftarrow Q.\texttt{pop}()$.
  if $\texttt{LP}(\mathcal{B}) < R^{\text{OPT}}$ **then**
    continue.{prune $\mathcal{B}$}.
  end if
  $\vec{B}^* \leftarrow \vec{B}^{\texttt{LP}}(\mathcal{B})$ {most promising $\vec{B} \in \mathcal{B}$}
  if Oracle$(\vec{B}^*) > L^*$ **then**
    Update $L^* \leftarrow$ Oracle$(\vec{B}^*)$ and $\vec{B}^{\text{OPT}} \leftarrow \vec{B}^*$.
  end if
  Branch: Partition $\mathcal{B} = \mathcal{B}_1 \cup \mathcal{B}_2$
  $Q \leftarrow Q \cup \{\mathcal{B}_1, \mathcal{B}_2\}$.
end while
return $\vec{B}^*$.

G No-Regret Guarantee for the Mitosis Algorithm

In the MAB framework each arm represents a candidate budget allocation $\vec{B}$ from the feasible set

$$\mathcal{B}_0 \triangleq \Big\{\vec{B} \in \mathbb{N}^K : \sum_{k=1}^{K} B_k \leq B\Big\}.$$

Pulling an arm $\vec{B}$ corresponds to calling the fast oracle $\text{Oracle}_{\text{small}}(\vec{B})$ (with $\texttt{epoch} = 1$) which returns a noisy estimate of the reward $\mu(\vec{B})$. Thus, by running a Multi-Armed Bandit (MAB) algorithm over the arms $\vec{B} \in \mathcal{B}_0$ we aim to select the arm with the highest expected reward without having to estimate $\mu(\vec{B})$ for every $\vec{B}$.

Definition (Reward Regret)**.** Let

$$\mu^\star \triangleq \max_{\vec{B}\in\mathcal{B}_0} \mu(\vec{B})$$

and denote by $\vec{B}_t$ the budget allocation (arm) chosen at time t. Then the instantaneous regret at time t is

$$\Delta_t \triangleq \mu^\star - \mu(\vec{B}_t),$$

and the cumulative (reward) regret over a time horizon T is defined as

$$R(T) \triangleq \sum_{t=1}^{T} \Delta_t = \sum_{t=1}^{T}\Big(\mu^\star - \mu(\vec{B}_t)\Big).$$

The goal is to design an algorithm whose cumulative regret grows sublinearly in T; that is, $\frac{R(T)}{T} \to 0$ as $T \to \infty$. In our setting, the optimal budget allocation $\vec{B}^\star$ (with $\mu(\vec{B}^\star) = \mu^\star$) will be identified as T increases.

Theorem 3. [No-Regret of the Mitosis Algorithm] Let $\mathcal{A}$ denote the set of arms that have been pulled. After running the algorithm for T rounds, the cumulative regret

$$R(T) \triangleq \sum_{t=1}^{T} (\mu^\star - \mu_t)$$

satisfies

$$R(T) = \sum_{\vec{B}\in\mathcal{A}} \mathbb{E}[N_T(\vec{B})]\,\Delta(\vec{B}) = O\left(\sum_{\vec{B}\in\mathcal{A}} \frac{\log T}{\Delta(\vec{B})}\right),$$

which matches the UCB1 regret bound.

Proof. The proof is built on the classical UCB1 analysis of Auer *et al.* [2002]. In the Mitosis Algorithm (Algorithm 1), each arm $\vec{B}$ is initialized with its upperbound $\texttt{LP}(\vec{B})$. The algorithm maintains two types of arms:

- **Unpromising arms:** Arms that are encapsulated in StemArm, who has not yet been pulled. Their index is given by $\texttt{LP}(\vec{B})$.
- **Candidate arms:** Arms that have been pulled at least once. For these, the UCB index at time t is defined as

$$I_t(\vec{B}) = \hat{\mu}_t(\vec{B}) + c\sqrt{\frac{\log t}{N_t(\vec{B})}},$$

where $\hat{\mu}_t(\vec{B})$ is the empirical mean, $N_t(\vec{B})$ is the number of pulls, and $c > 0$ is a constant.

The algorithm runs for T rounds; by the end, let $\mathcal{A}$ denote the final set of candidate arms. We consider two cases based on the location of the optimal arm $\vec{B}^\star$.

Case 1: $\vec{B}^\star \in \mathcal{A}$. In this case, the optimal arm has been pulled at least once. Therefore, the candidate arms $\mathcal{A}$ form a sub-MAB instance where we can directly apply UCB1's regret bound on; the arms in StemArm are not pulled anyway, so they do not contribute to regret. Standard UCB1 analysis (using a peeling argument and concentration inequalities, see Auer *et al.* [2002]) shows that the expected pulls on any suboptimal arms, denoted by $N_t(\vec{B})$, satisfy

$$\mathbb{E}[N_t(\vec{B})] = \mathcal{O}\left(\frac{\log t}{\Delta(\vec{B})^2}\right). \tag{35}$$

Thus, the regret incurred by arms in $\mathcal{A}$ is

$$R(T) = \sum_{\vec{B}\in\mathcal{A}} \mathbb{E}[N_T(\vec{B})]\,\Delta(\vec{B}) = \mathcal{O}\left(\sum_{\vec{B}\in\mathcal{A}} \frac{\log T}{\Delta(\vec{B})}\right).$$

Case 2: $\vec{B}^\star \notin \mathcal{A}$. We prove that this case will never happen by contradiction. In other words, the optimal arm that represents the optimal budget solution for CBB will also be budded out by the StemArm in Mitosis algorithm, as the time horizon is sufficiently large.

Let $\vec{B}^{2nd} := \arg\max_{a\in\mathcal{A}} \mu(a)$ be the arm with the highest mean in the candidate arms. Since the number of pulls for all other suboptimal arms satisfies (35), the number of pulls for $\vec{B}^{2nd}$ grows linearly with t:

$$\mathbb{E}[N_T(\vec{B}^{2nd})] = T - \mathcal{O}(\log T).$$

Since $\vec{B}^\star \in$ StemArm, by the end of the algorithm, the StemArm's index ($\texttt{LP}$(StemArm))is smaller than the UCB index of $\vec{B}^{2nd}$. Since $\vec{B}^\star \in$ StemArm, we have

$$\texttt{LP}(\text{StemArm}) \geq \texttt{LP}(\vec{B}^\star) \geq \mu(\vec{B}^\star).$$

then, the event that the suboptimal arm $\vec{B}^{2nd}$'s UCB index be strictly greater than the optimal arm's mean $\mu(\vec{B}^\star)$: for $\mu(\vec{B}^\star) > \hat{\mu}_T(\vec{B}^{2nd})$,

$$\mathbf{Pr}\left[I_T(\vec{B}^{2nd}) \geq \mu(\vec{B}^\star)\right] \tag{36}$$

$$= \mathbf{Pr}\left[\hat{\mu}_T(\vec{B}^{2nd}) + c\sqrt{\frac{\log(\mathcal{O}(T))}{N_T(\vec{B}^{2nd})}} \geq \mu(\vec{B}^\star)\right] \tag{37}$$

$$\leq \exp\left\{-\frac{\mathcal{O}(T)(\mu(\vec{B}^{2nd}) - \mu(\vec{B}^\star))^2}{c'}\right\} \tag{38}$$

$$\tag{39}$$

The probability declines exponentially. Since the probability of the event in Case 2 decays exponentially with T, its contribution to the overall expected regret is negligible compared to the regret in Case 1. In other words, with probability tending to one as $T \to \infty$, the optimal arm $\vec{B}^\star$ is eventually pulled and becomes a candidate arm. Therefore, the overall expected regret of the Mitosis algorithm is dominated by the regret incurred in Case 1, and we have

$$R(T) = \mathcal{O}\left(\sum_{\vec{B} \in \mathcal{A}} \frac{\log T}{\Delta(\vec{B})} \right).$$

Overall the regret of Mitosis is controlled by the classical UCB1 guarantee, up to a constant factor, and hence the algorithm achieves near-optimal performance. This completes the proof.

$\square$